ALGEBRA UNIT 11-GRAPHING QUADRATICS

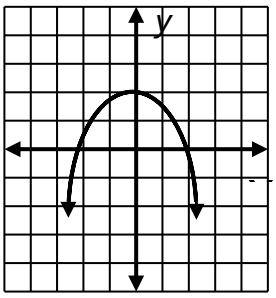
THE GRAPH OF A QUADRATIC FUNCTION (DAY 1)

- The **Quadratic Equation** is written as: _____ ; this equation has a degree of _____.
  - Where **a, b** and **c** are integer coefficients (where a ≠ 0)
- The graph of this equation is called a _____; it is _____.
- Parabolas are functions because they _____.

2 TYPES OF PARABOLA SHAPES

When **"a"** is **positive,** the parabola opens: _____ Where the curve reaches a _____	

When **"a"** is **negative,** the parabola opens: _____ Where the curve reaches a _____	



Draw in the line of symmetry of the parabola on the grid above, left.

This line of symmetry is called the _____

- It is always a vertical line that goes through the turning point of the curve.

Formula:

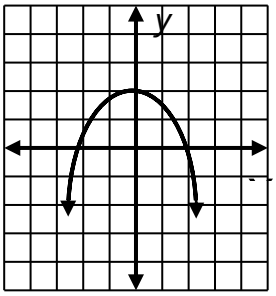
Axis of Symmetry:

Examples:

1. What is an equation of the axis of symmetry of the parabola represented by $y = -x^2 + 6x - 4$?

2. What is the equation for the axis of symmetry for : $y = -x^2 - 2x - 1$

   (1) $x = 1$ (2) $x = -1$ (3) $y = 1$ (4) $y = -1$



Turning Point: Is another term for the **vertex** of the parabola. The "vertex" has the coordinates of (x, y).

To Find Turning Point (T.P.)

-
-

Zeros (roots) of the equation are the points where the parabola _____ the x – axis, so y = _____.

How to use the zeros to write a QUADRATIC equation:

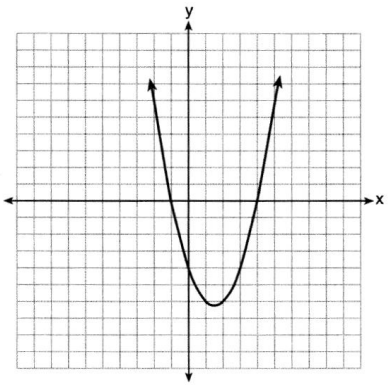
Write a quadratic equation that has zeros that are -5 and 8.

Examples:

1. What are the coordinates of the turning point of the parabola whose equation is $y = x^2 - 4x + 4$?

   (1) (2, 0) (2) (–2, 16) (3) (2, –4) (4) (–2, 0)

2. What are the zeros of the parabola on the grid? Write an equation for this graph.



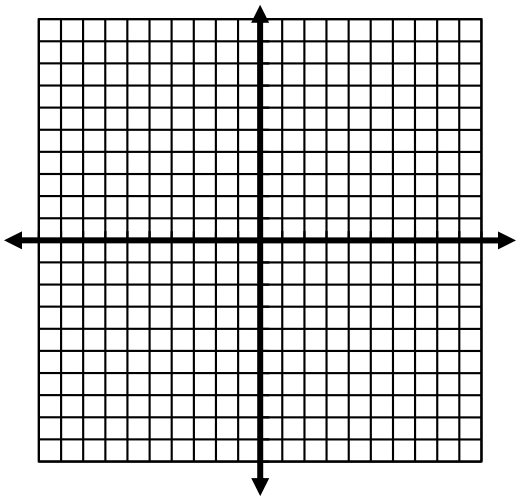
3. If the roots of a quadratic equation are –2 and 3, the equation can be written as:

   1) (x - 2)(x + 3) = 0
   2) (x + 2)(x - 3) = 0
   3) (x + 2)(x + 3) = 0
   4) (x - 2)(x - 3) = 0

GRAPHING QUADRATIC FUNCTIONS (DAY 2)

How to Graph Parabolas:

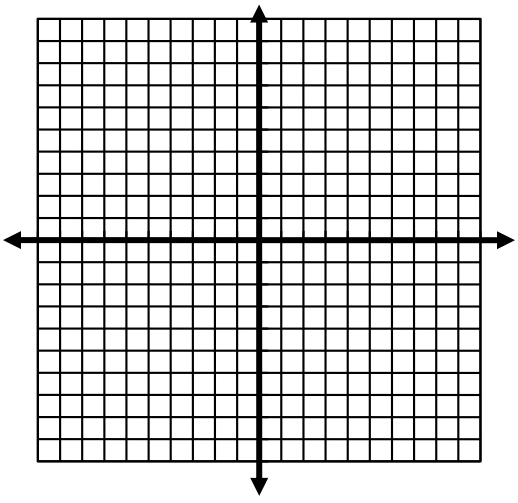
1. Find the **axis of symmetry** by using the formula.
2. Substitute the x-value back into the equation to find the **turning point and describe it as a <u>max</u> or <u>min</u> pt.**
3. Make a **table** of values.
4. **Graph** the points.



EX1: GRAPH: $y = x^2 - 4x$

State the following: Domain: _____ Range: _____

Is the above graph a function?, Explain. _____



EX2: GRAPH: $f(x) = -x^2 - 2x + 3 \ (-4 \le x \le 2)$ ← ***<u>This is called an interval</u>, which means your table should cover the x values of -4 to 2.***

State the following: Domain: _____ Range: _____

Is the above graph a function? Explain. _____

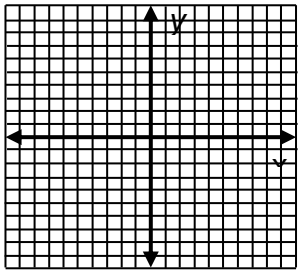
EXPLORING THE GRAPHED QUADRATIC EQUATION (DAY 3)

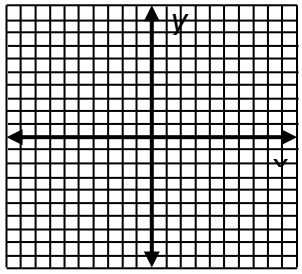
Quadratic functions are written in the form: _____

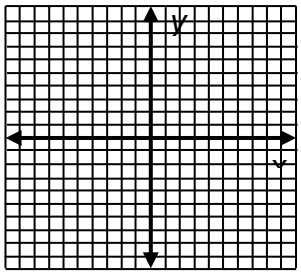
The **x – intercepts** (when **y = 0**) of the parabola $y = ax^2 + bx + c$ are called the _____ or _____ of the equation $(ax^2 + bx + c = 0)$

How many roots are possible to obtain from a quadratic equation? _____

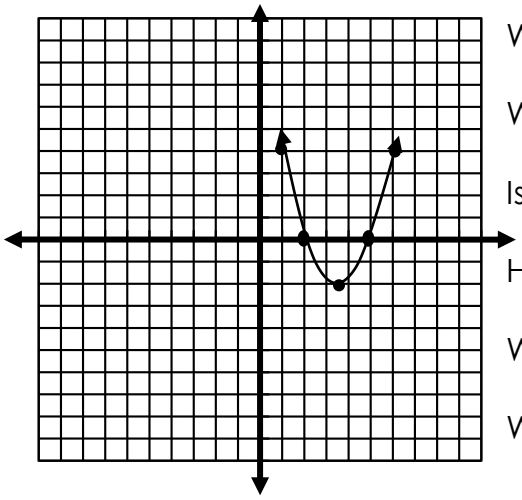
Draw a picture to illustrate each situation







EX1. Given the following graph of the equation $y = x^2 - 7x + 10$. Answer the following questions.



What is the axis of symmetry? _____

What are the coordinates of the turning point?_____

Is the T.P. a max or minimum point? _____

How many zeros are there?_____

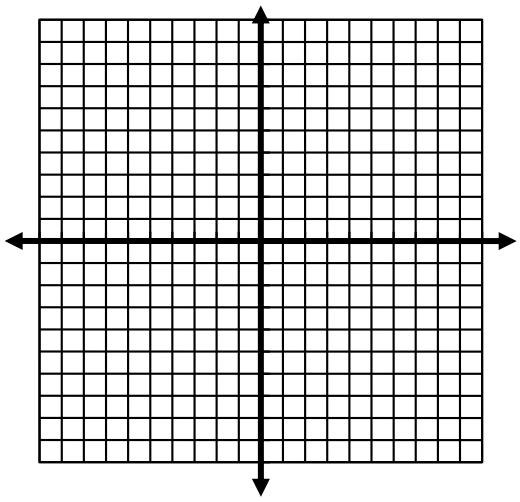
What are the solutions of this equation? _____

What are the solutions called? _____

Now, solve the equation algebraically: $0 = x^2 - 7x + 10$

What do you notice?

Ex2: GRAPH: $f(x) = -x^2 - 6x - 5$



What is the axis of symmetry? _____

What are the coordinates of the turning point?_____

Is the T.P. a max or minimum point? _____

How many zeros are there?_____

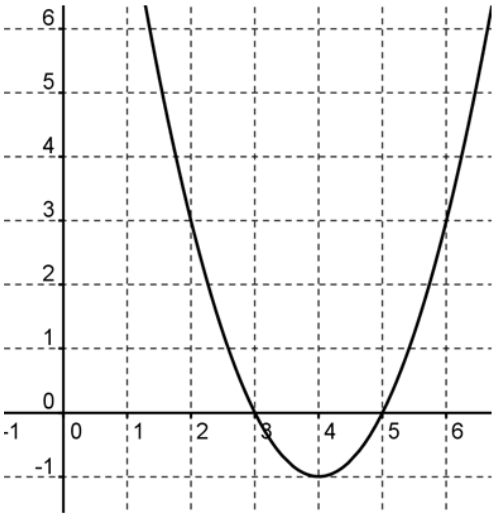
What are the solutions of this equation? _____

What do you call these solutions? _____

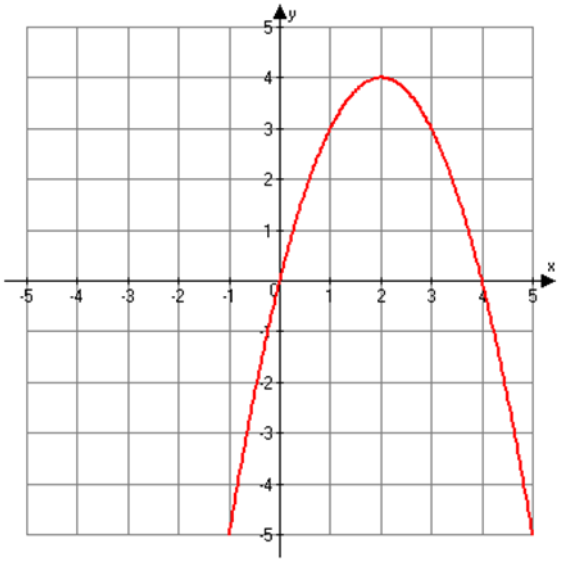
Procedure for writing an equation for a graphed quadratic function

Given the two graphs below, write an equation for each.

Ex 3:

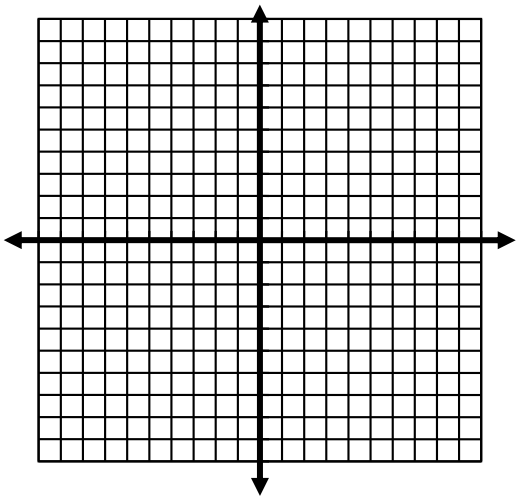


Ex 4:



GRAPHING/EXPLORING QUADRATIC EQUATIONS CONT… (DAY 4)

Ex1: Graph: $f(x) = 3x^2 + 6x - 4$



Axis of symmetry: _____

Vertex: _____

Domain _____

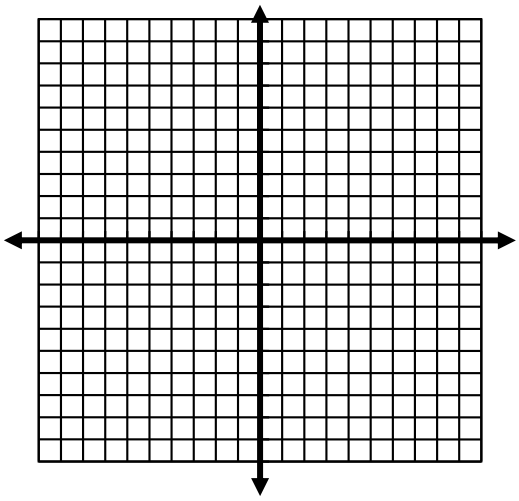
Range _____

Find f(1) _____

Find f(-2) _____

Is the parabola a maximum or a minimum? Explain: _____

Ex2: Graph: $f(x) = -2x^2 - 8x - 2$



Axis of symmetry: _____

Vertex: _____

Domain _____

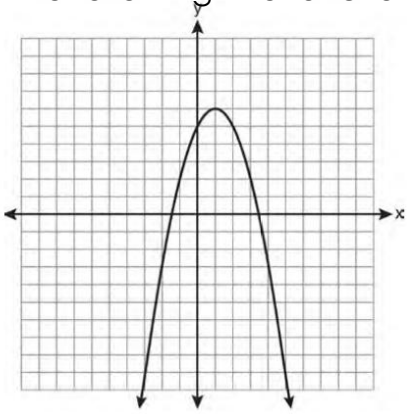
Range _____

What is the y-intercept _____

Find f(0) _____

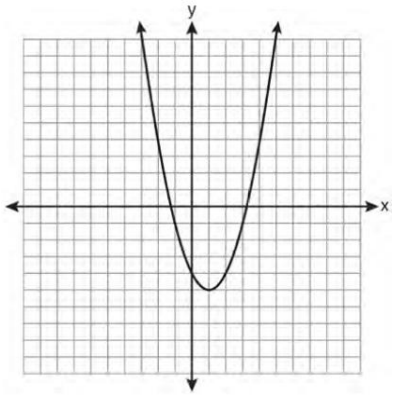
Find f(x) = 4 _____

Ex3: Given the following two functions, which one has the larger maximum?



$f(x) = -2x^2 - 8x + 3$

Ex4: Given the following three functions, which one has the least minimum?



$y = x^2 + 4x + 4$ $f(x) = x^2 + 2x - 24$

Ex5: The equation of the axis of symmetry of the graph of $y = 2x^2 - 3x + 7$ is

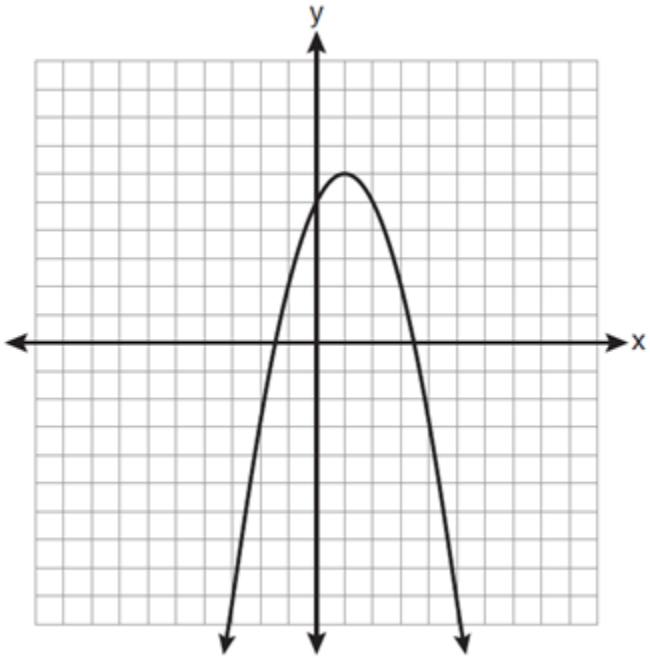
(1) $x = \frac{3}{4}$ (2) $y = \frac{3}{4}$ (3) $x = \frac{3}{2}$ (4) $y = \frac{3}{2}$

Ex6: The roots of the equation $3x^2 - 27x = 0$ are

(1) 0 and 9 (3) 0 and -9

(2) 0 and 3 (4) 0 and -3

Ex7: What are the vertex and axis of symmetry of the parabola shown in the graph below?



(1) Vertex (1, 6); axis of symmetry: y = 1

(2) Vertex (1, 6); axis of symmetry: x = 1

(3) Vertex (6, 1); axis of symmetry: y = 1

(4) Vertex (6, 1); axis of symmetry: x = 1

SOLVING QUADRATIC – LINEAR SYSTEMS (DAY 5)

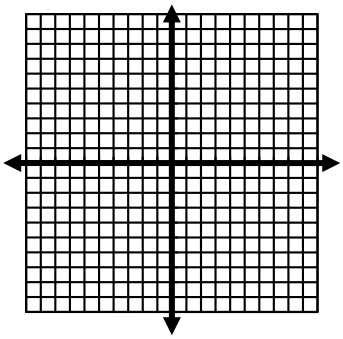
Two equations will be given to you with the directions to solve the system graphically.

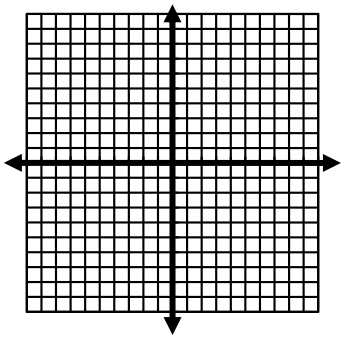
- One equation will be a quadratic. This equation has degree _____
- The second equation will be linear. This equation has degree _____

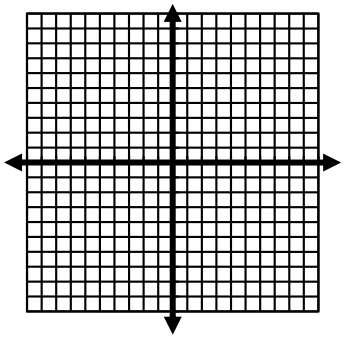
You will have to create a table of values for the quadratic equation and graph the linear equation using _____.

Where the two graphs _____, this is your _____.

There are three possible situations as answers illustrated below. Indicate the number of solutions in each representation.





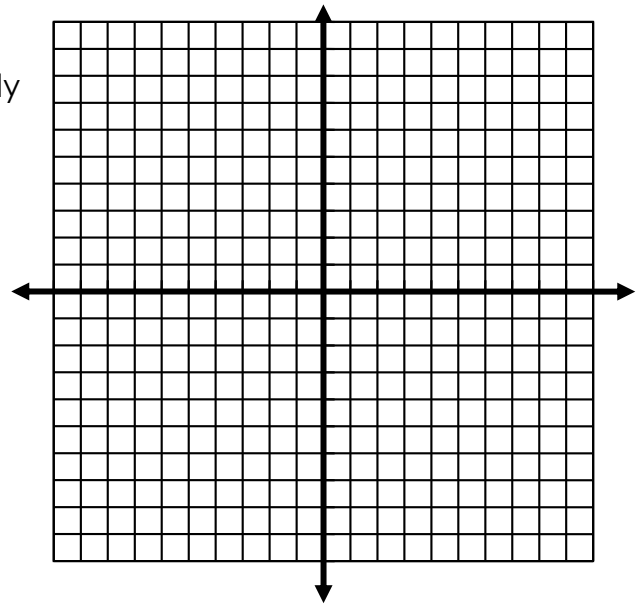


Examples:

1. Solve the following system of equations graphically

$y = -x^2 + 4x - 3$

$x + y = 1$



To check on your graphing calculator (find intersection):

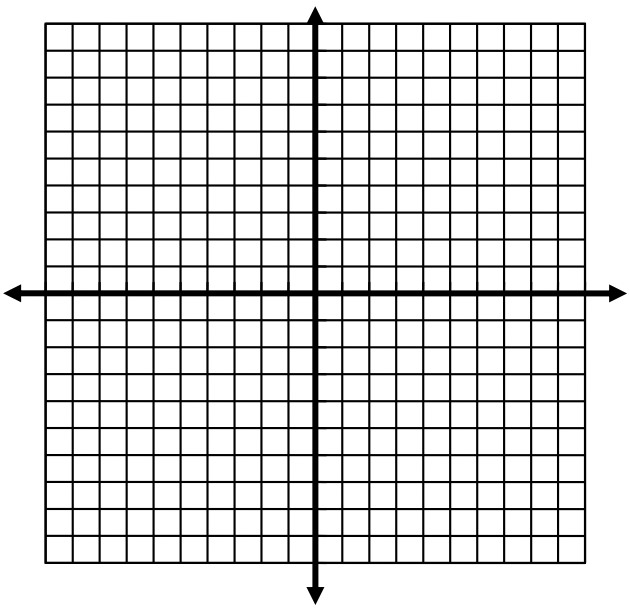
1) Go to [2nd] [Trace] (Calculate) and pick [#5] (intersection)

2) Move cursor to wanted intersection point and hit [Enter] [Enter] [Enter]

2. Solve the following system of equations graphically and check.

$y = x^2 + 4x + 4$

$y = -2x + 4$



3. The graphs of the equations $y = x^2$ and $x = 2$ intersect in:

(1) 1 point (2) 2 points (3) 3 points (4) 4 points

4. Which is a solution or the following system of equations?

$y = 2x - 15$

$y = x^2 - 6x$

(1) (3, –9) (2) (0, 0) (3) (5, 5) (4) (6, 0)

5. When the graphs of the equations $y = x^2 - 5x + 6$ and $x + y = 6$ are drawn on the same set of axes, at which point do the graphs intersect?

(1) (4, 2) (2) (5, 1) (3) (3, 3) (4) (2, 4)

SOLVING QUADRATIC – LINEAR SYSTEMS ALGEBRAICALLY (DAY 6)

PROCEDURE FOR SOLVING QUAD-LINEAR SYSTEMS ALGEBRAICALLY:

1. Make sure both equation are in y = form if necessary
2. Substitute the linear equation into the 'y part' of the quadratic equation, to have only one variable left to solve in the equation.
3. Get NEW quadratic equation into standard form (_____) and _____
4. **Since it is a quadratic:** Must **FACTOR TO SOLVE FOR X**. (How many answers should you get?_____)
5. Must find other variable (y) by substituting your 'x' answers into one of the equation and solve for y.
6. Check solutions

Examples:

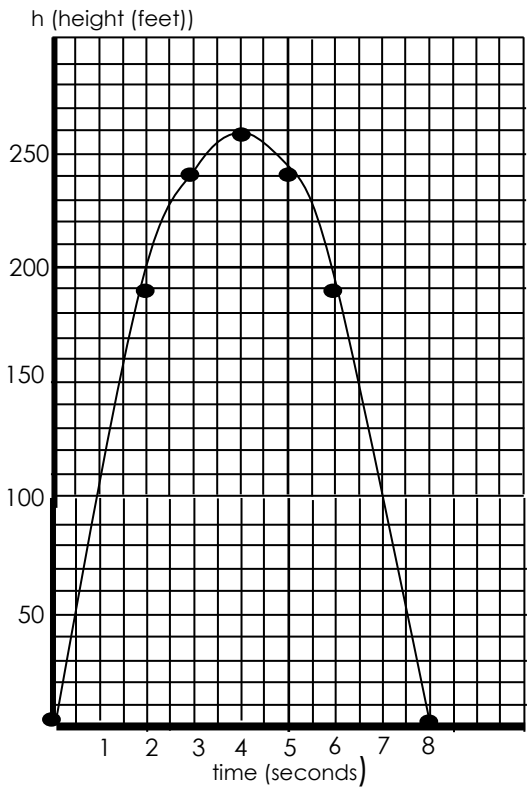
1. Solve the following system: $y = x^2 - x + 2$, $y = 2x$

2. Find the solutions of: $y = -x^2 + 4x - 3$
$x + y = 1$

3. Solve for the solutions: $y = x^2 - 7x + 13$
$x - y = 2$

APPLICATIONS WITH PARABOLIC FUNCTIONS (DAY 7)

EX. 1 Using the graph at the right, It shows the **height *h*** in feet of a small rocket ***t* seconds** after it is launched. The path of the rocket is given by the equation:
$h = -16t^2 + 128t$.



1. How long is the rocket in the air? _____

2. What is the greatest height the rocket reaches? ____

3. About how high is the rocket after 1 second? _____

4. After 2 seconds,
   about how high is the rocket?_____
   is the rocket going up or going down? _____

5. After 6 seconds,
   about how high is the rocket? _____
   is the rocket going up or going down? _____

6. Do you think the rocket is traveling faster from 0 to 1 second or from 3 to 4 seconds? Explain your answer.

7. Using the equation, find the **exact** value of the height of the rocket at 2 seconds.

8. What is the domain of the graph?

9. What is the range of the graph?

10. Express the interval over which the graph is increasing.

11. Express the interval over which the graph is decreasing.

EX2: A ball is thrown in the air. The path of the ball is represented by the equation $h = -t^2 + 8t$. Graph the equation over the interval $0 \le t \le 8$ on the accompanying grid.

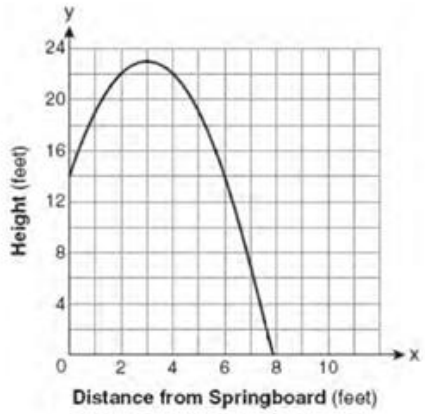
height (meters)

time (seconds)

a) What is the maximum height of the ball?_____

b) What is the amount of time that the ball is above 7 meters? _____

EX3: A swim team member performs a dive from a 14-foot high springboard. The parabola below shows the path of her dive.



a) What is the axis of symmetry? _____

b) Find f(6) _____

EX4: Consider the graph of the equation $y = ax^2 + bx + c$, when $a \neq 0$. If a is multiplied by 3, what is true of the graph of the resulting parabola?

1) The vertex is 3 units above the vertex of the original parabola.
2) The new parabola is 3 units to the right of the original parabola
3) The new parabola is wider than the original parabola.
4) The new parabola is narrower than the original parabola.

EX5: Melissa graphed the equation $y = x^2$ and Dave graphed the equation $y = -3x^2$ on the same coordinate grid. What is the relationship between the graphs that Melissa and Dave drew?

EX6: The graph of a parabola is represented by the equation $y = ax^2$ where a is a positive integer. What happens to the new parabola if a is multiplied by 2? What if multiplied by $\frac{1}{2}$?

QUADRATIC APPLICATION WORD PROBLEMS (SOLVING ALGEBRAICALLY) (DAY 8)

Warm-Up: If 5 is a root of $x^2 - 3x + k = 0$, find k.

What is the other root?

Procedure for Word Problems

- Highlight given functions in the word problems
- Identify variables in the problem/function and highlight what they represent
- READ question carefully to determine WHAT variable needs to be solved for

1. After *t* seconds, a ball tossed in the air from the ground level reaches a height of *h* feet given by the function $h(t) = 144t - 16t^2$.

   a. What is the height of the ball after 3 seconds?

   b. What is the maximum height the ball will reach?

   c. After how many seconds will the ball hit the ground before rebound?

2. A rocket carrying fireworks is launched from a hill 80 feet above a lake. The rocket will fall into the lake after exploding at its maximum height. The rocket's height above the surface of the lake is given by the function $h(t) = -16t^2 + 64t + 80$.

   a. What is the height of the rocket after 1.5 seconds?

   b. What is the maximum height reached by the rocket?

   c. After how many seconds after it is launched will the rocket hit the lake?

3. A rock is thrown from the top of a tall building. The distance, in feet, between the rock and the ground t seconds after it is thrown is given by $d(t) = -16t^2 – 4t + 382$. How long after the rock is thrown is it 370 feet from the ground?